# Grand Unified Theories in Cosmology

J. Ellis

| **Email alerting service** | Receive free email alerts when new articles cite this article - sign up in the box at the top right-hand corner of the article or click  **here** |
| --- | --- |

# Grand unified theories in cosmology

By J. Ellis†

*All Souls' College and Department of Theoretical Physics, Oxford, U.K.*

This paper reviews grand unified theories and some of their possible applications to cosmology. The philosophy of grand unification to be followed is first developed, some low-energy tests described, and then expectations for new interactions causing baryon decay and neutrino masses are presented. The experimental situations concerning these two possibilities are briefly reviewed. A discussion is given of the possible relevance of baryon-number violating reactions in grand unified theories to understanding the problem of baryosynthesis, and a possible connection with the neutron electric dipole moment is mentioned. Possible interfaces between cosmology and particle physics involving neutrinos are mentioned.

## 1. Introduction

This paper discusses several of the interfaces between cosmology and grand unified theories (GUTs) of elementary particle physics (Ellis 1981 *a*, *b*; Langacker 1981 *a*, *b*; Nanopoulos 1980). The two main connections discussed here are the mechanism suggested by baryon-number violating interactions in GUTs for explaining the baryon–antibaryon asymmetry apparent in the Universe, and the likelihood in the context of GUTs that neutrinos have masses that might be large enough to influence the formation and structure of galaxies. Some remarks will also be made about limits imposed by cosmology and particle physics on the masses and number of different neutrino types (Schramm, this symposium), and on the possible existence of other massive, stable, weakly interacting neutral particles, the 'nuinos' of supersymmetric theories. The important topics of phase transitions in the early Universe and the abundance of the magnetic monopoles expected in GUTs are discussed by Guth (this symposium).

As a preliminary to examining these different applications of GUTs to cosmology, the §2 of this paper reviews how one is led to the speculative idea of grand unification from the 'standard model' of elementary particle physics, and how baryon- and lepton-number changing interactions giving rise to baryon decay and neutrino masses appear naturally in GUTs. The problem of generating the baryon–antibaryon asymmetry of the Universe is posed in §3, and it is shown how GUTs meet the Sakharov criteria for baryosynthesis (Sakharov 1967). A possible connection (Ellis *et al.* 1981 *a*, *b*, *c*) between the observed baryon:photon ratio and an elementary particle physics observable, the neutron's electric dipole moment, is also discussed. The cosmological aspects of neutrinos and nuinos are discussed in §4, and some speculations about the far future of the Universe are summarized in §5.

† Permanent address: Theory Division, CERN, CH-1211 Geneva 23, Switzerland.

## 2. FROM THE STANDARD MODEL TO GRAND UNIFICATION

There is now a generally accepted 'standard model' of elementary particle physics, which incorporates hadrons as composite states made out of quarks, and leptons. As far as we can probe experimentally today, both quarks and leptons appear point-like and structureless when observed at high energies, corresponding by the uncertainty principle to very short distances of order $10^{-16}$ cm. We distinguish three types of fundamental interactions between these quarks and leptons, all of which are now believed to be described by gauge theories. Hence they involve the interaction of elementary fermions with gauge bosons of spin 1, as indicated in figure 1.
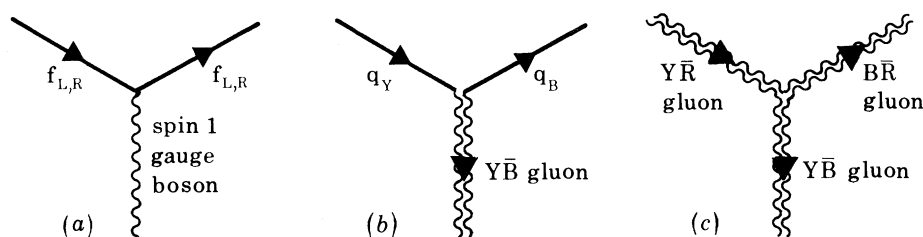


FIGURE 1. The basic interactions of gauge theories. (a) A spin-one gauge boson interacting with a fermion conserves helicity. Gluons carry colour and hence can (b) change the colour of a quark, or (c) have self-interactions.

The strong interactions are viewed (Marciano & Pagels 1978) as originating from a gauging of a concealed SU(3) symmetry. Each species (or flavour) of quark exists in three varieties (or colours) with identical weak and electromagnetic interactions, and the SU(3) group is that of rotations in the three-dimensional complex space populated by these 'colours'. The resulting gauge theory is called Quantum Chromodynamics (QCD), and contains eight vector gluons analogous to the photon, which, however, carry 'colour' themselves and hence can change the colours of quarks and also interact among themselves, as indicated in figure 1b, c. The familiar strong nuclear interactions are nowadays viewed as residual van der Waals forces between hadrons with no net colour, just as conventional molecular forces act between systems with no net electromagnetic charge.

The weak and electromagnetic interactions are generally believed to be unified into a gauge theory that at least approximates the minimal $SU(2) \times U(1)$ model (Glashow 1961; Weinberg 1967; Salam 1968) at low energies. Parity is violated intrinsically in this model because left-handed fermions ($\nu, l_L^-, q_L$) are assigned to doublets of SU(2) whereas right-handed fermions ($l_R^-, q_R$) are singlets of SU(2) and hence do not feel the charged weak current interactions. It is often convenient to take account of the right-handed fermions by focusing on their left-handed antifermion conjugates ($l_L^+, \bar{q}_L$), in which case we see that charge conjugation (C) is also intrinsically violated.

All confirmed experimental results conform to the predictions of this 'standard model'. QCD has many qualitative and semi-quantitative successes such as understanding the lightness of the pion and other aspects of hadron spectroscopy, the qualitative features of deep inelastic scattering at large momentum transfers, and the annihilation of $e^+e^-$ into two-jet (interpreted as quark–antiquark pairs) and three-jet (interpreted as q–$\bar{q}$–gluon) final states. On the other hand, we do not yet have a precision test of QCD as striking as the verification to many decimal places of the QED predictions for the anomalous magnetic moments of the electron and muon.

The Glashow–Weinberg–Salam model describes accurately all known weak interaction data. For example, dozens of weak neutral current experiments are well fitted (Kim *et al.* 1981; Liede & Roos 1980) by two parameters, the neutral weak mixing angle $\theta_W$ and the overall strength $\rho\{\alpha\sqrt{(\sigma_{NC}/\sigma_{CC})}\}$:

$$\sin^2\theta_W = 0.20 \text{ to } 0.25; \quad \rho = 0.95 \text{ to } 1.05, \tag{1}$$

as we will see in more detail later on.

Despite its successes, the 'standard model' has many egregious shortcomings. For example, it has no explanation for charge quantization: why is $|Q_e|/|Q_p| = 1$ with a precision of perhaps 20 decimal places? If this ratio were not unity, the electrostatic repulsion between galaxies would overwhelm their gravitational attraction. Furthermore, the 'standard model' has no explanations for the fundamental fermion masses and charged weak mixing angles $\theta_c$ (Kobayashi & Maskawa 1973). It even has three independent gauge coupling constants,

$$g_3 \neq g_2 \neq g_1, \tag{2}$$

corresponding to the different SU(3), SU(2) and U(1) group factors. Indeed, a simple parameter count reveals at least 20 parameters in the 'standard model': three gauge couplings, two non-perturbative vacuum angles, at least six quark masses and three charged lepton masses, at least four charged current mixing angles and phases, and two parameters to describe the masses of the weak bosons. Clearly, one would like to reduce the number of parameters, and the first step proposed in GUTs is to combine the known interactions into a single semi-simple gauge group G which is then broken down to the known low-energy theories in at least two steps:

$$G \xrightarrow[10^{15}\,\text{GeV}]{} SU(3) \times SU(2) \times U(1) \xrightarrow[10^2\,\text{GeV}]{} SU(3) \times U(1). \tag{3}$$

Such a theory would have a unique gauge coupling. The second stage of symmetry breaking takes place at a mass scale close to that of the expected but as yet unobserved intermediate weak vector bosons $W^\pm$ and $Z^0$. We shall see shortly why the first stage of symmetry breaking in (3) is so high; first let us explore further the motivations for the grand unification philosophy being adopted.

Fermions are generally grouped into 'generations' with similar masses ($m_q \sim m_{q'} \sim m_l$) and small charged weak mixing angles between different generations. The conventional generation assignments are shown in figure 2. All stable matter in the Universe is composed of fermions in the first generation, and it is a mystery why 'Xerox' copies exist, and indeed how many exist with higher masses than those seen so far. The existence of a generation pattern is not the artefact of an overactive Gestalt mechanism: the Monte Carlo generation of random quark and lepton masses has confirmed that the fermions within each generation are correlated in a statistically significant way, and showed that in a random model one would not expect the generalized charged weak mixing angles to be small (Froggatt & Nielsen 1979).

We shall adopt a philosophy of grand unification that embodies the generation structure of fundamental fermions by unifying the interactions of the particles within each generation. In this way we get direct quark–lepton and quark–antiquark interactions that lead to baryon decay. The problem of predicting the number of generations is left to another generation of physicists to solve, though later on we shall meet a couple of phenomenological constraints on the number.

The main obstacle to be overcome within this philosophy of grand unification is the inequality of the different fundamental coupling strengths:

$$g_3 \gg g_2, g_1, \tag{4}$$

reflecting the fact that the strong interactions are strong while the weak are weak! All the couplings (4) should be equal when grand unification is achieved. A way round this obstacle is offered by the renormalization group (Stueckelberg & Peterman 1953; Gell-Mann & Low 1954), which teaches us that coupling 'constants' in fact vary with the scale of the energy $Q$ at which they are being measured. In particular, the strong interactions described by QCD
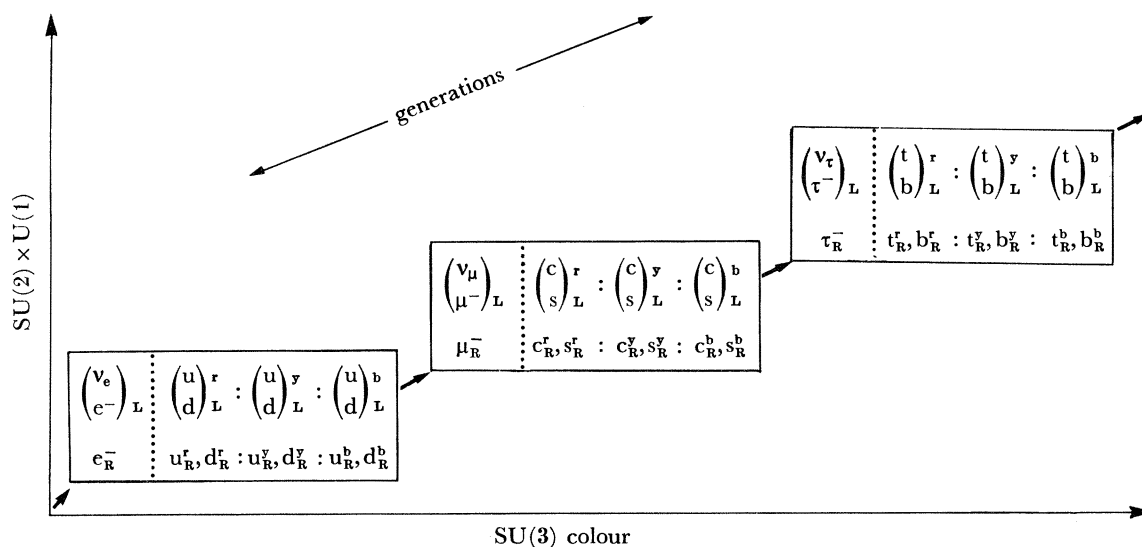


FIGURE 2. The apparent 'generation' structure of fundamental fermions. The horizontal axis corresponds to SU(3) colour properties, the vertical axis to SU(2) × U(1) representation contents.

have the famous 'asymptotic freedom' property (Politzer 1973; Gross & Wilczek 1973) that they get weaker at higher energies:

$$\alpha_3(Q^2) \equiv \frac{g_3^2(Q^2)}{4\pi} = \frac{12\pi}{(33-2N_q)\ln Q^2/\Lambda^2} + \cdots, \tag{5}$$

where $N_q$ is the number of quarks with masses less than $Q$ and $\Lambda$ is a fundamental scale parameter for the strong interactions, which presumably lies between 0.1 and 1 GeV. This property of asymptotic freedom (5) is essential to understanding the simplicity revealed in deep inelastic lepton–hadron scattering experiments at high energies. The other, weak, couplings also vary with increasing energy but at somewhat slower rates, so that the overall effect is to make the strong and weak coupling strengths approach each other (Georgi et al. 1974); for example

$$\frac{1}{\alpha_3(Q^2)} - \frac{1}{\alpha_2(Q^2)} = -\frac{11}{12\pi}\ln\left(m_X^2/Q^2\right) + \cdots, \tag{6}$$

where $m_X$ is the conjectured 'grand unification scale' at which all the couplings are to become equal, corresponding to the vanishing of the logarithm in (6). The resulting picture for the development of the different couplings (4) is shown in figure 3.

It is easy to deduce from the logarithmic approach (6) that there will be an exponential ratio between the scale $\Lambda$ at which the strong interactions become strong and the grand unification scale $m_X$:

$$m_X/\Lambda = \exp\left(O(1)/\alpha + O(1) \times \ln \alpha + O(1)\alpha^0 + \dots\right), \tag{7}$$

where $\alpha$ is the conventional fine structure constant. Putting the experimental value of $\alpha = \frac{1}{137}$ into (7) one finds that $m_X = O(10^{14} \text{ to } 10^{15})$ GeV (Ellis *et al.* 1980). This scale may seem astronomically high, but is in fact several orders of magnitude below the scale of the Planck mass $m_P = G_N^{-\frac{1}{2}} \approx 10^{19}$ GeV, which is the scale at which quantum gravitational effects must become important. It is therefore not obviously inconsistent to neglect gravitation while one is unifying the other fundamental interactions.
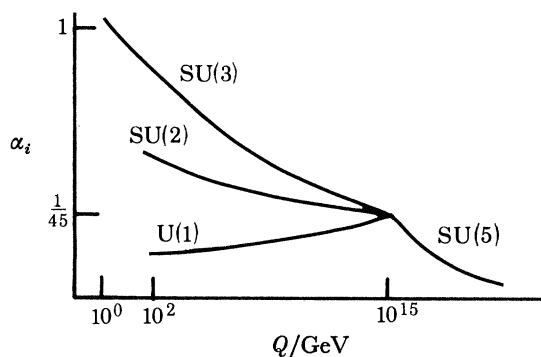


FIGURE 3. A sketch of the manner in which the SU(3), SU(2) and U(1) couplings are supposed to come together in a GUT.

In view of the exponential ratio (7), this is a non-trivial conclusion: indeed the grand unification philosophy outlined above only makes sense if $m_X < O(10^{19})$ GeV (otherwise we would have to include gravity) and $m_X > O(10^{14})$ GeV (otherwise the proton would decay faster than the observed limit, and furthermore it would be difficult to generate the matter–antimatter asymmetry in the early Universe, as we shall see later). Feeding these bounds on $m_X$ into (7) we can infer non-trivial constraints on the range of $\alpha$ that do not give nonsensical values of $m_X$ (Ellis & Nanopoulos 1981):

$$\frac{1}{120} > \alpha > \frac{1}{170}. \tag{8}$$

The fact that the observed value of the fine structure constant lies within this range may be a broad hint that the grand unification philosophy is correct.

Let us now turn to some simple models for grand unification. They should be based on semi-simple groups of rank at least four – the sum of the ranks of the SU(3), SU(2) and U(1) groups that we wish to include in our GUT. The only suitable (Georgi & Glashow 1974) group of rank four itself is SU(5), the group of special unitary rotations in a space with five complex dimensions. This theory contains 24 gauge vector bosons, nine of which are the familiar photon and gluons, and three of which are the intermediate vector bosons $W^\pm$ and $Z^0$ of the weak interactions, which we expect to have masses $O(80–90)$ GeV and hope to find in experiments at CERN in the next few years. The remaining 12 vector bosons are triplets of coloured particles X and Y, which also form a doublet of the weak SU(2) group, and their antiparticles $\overline{X}$ and $\overline{Y}$. These bosons will have masses $O(10^{15})$ GeV and 'carry' new hyperweak interactions, in close analogy to the way the $W^\pm$ and $Z^0$ 'carry' the conventional weak interactions. All

these 24 gauge bosons mediate interactions between quarks and leptons assigned to the three or more generations of figure 2. Each generation contains at least 15 helicity states, which are assigned to a reducible $\bar{5} + \mathbf{10}$ dimensional representation. As an example, one of the $\bar{5}$'s contains the first generation fermions

$$\underbrace{(\bar{d}_R, \bar{d}_Y, \bar{d}_B}_{\text{QCD SU(3)}}; \underbrace{e^-, \nu_e)_L,}_{\text{weak SU(2)}} \tag{9}$$

where the strong interactions act on the first three indices, the SU(2) weak interactions act on the last two indices, and the subscript L reminds us that we are dealing with left-handed helicity states. The new hyperweak interactions will interchange the first three and the last two indices, and hence change quarks into leptons and vice versa. Taken together with the transitions from quarks to antiquarks (or quark–quark annihilations) that occur within the ten-dimensional representation, which have not been exhibited explicitly, these hyperweak interactions can lead to baryon decay. An uncomfortable feature of contemporary gauge theories including GUTs is the requirement of spinless Higgs fields to cause spontaneous symmetry breakdown and give masses to some of the vector bosons. In the minimal SU(5) GUT we in fact need two multiplets of Higgses, a $\mathbf{24}$ to break SU(5) down to SU(3) × SU(2) × U(1) with a vacuum expectation value $\langle 0|\phi_{24}|0\rangle = O(10^{15})$ GeV giving masses to the X and Y bosons, and a $\mathbf{5}$ to break weak SU(2) × U(1) down to the exact electromagnetic U(1) group via a vacuum expectation value $\langle 0|H_5|0\rangle = O(10^2)$ GeV giving masses to the $W^\pm$, $Z^0$, quarks and leptons.

The next-to-minimal GUT is based on SO(10), the group of orthogonal rotations in a space of ten real dimensions (Georgi 1975). It contains 45 gauge bosons, thus in principle providing more ways for baryons to decay. Each generation of fundamental fermions is now assigned to a 16-dimensional irreducible representation that includes an extra colourless, neutral fermion, which is a candidate for a left-handed antineutrino, and enables one to generate the neutrino masses that will be met later on. Even the minimal version of SO(10) requires at least three irreducible Higgs representations: $\mathbf{10}$, $\mathbf{16}$ and $\mathbf{45}$. Larger GUTs tend to have even more complicated sets of Higgses and will not be discussed here, as SU(5) and SO(10) already possess most of the essential features.

Let us now turn to some predictions of these GUTs for quantities observable at low energies. One is charge quantization, which is in fact a feature common to all theories where electromagnetic charge is included in a semi-simple group. The sum of the charges of all the particles in any representation must add up to zero, and for example in the $\bar{5}$ of the SU(5) GUT we have

$$3Q_{\bar{d}} + Q_{e^-} = 0, \tag{10}$$

from which we deduce $Q_{\bar{d}} = +\frac{1}{3}$, $Q_d = -\frac{1}{3}$, $Q_u = +\frac{2}{3}$ and hence

$$Q_p = 2Q_u + Q_d = +1. \tag{11}$$

Thus we understand why galaxies are not pushed apart by electrostatic repulsion! We can also compute (Georgi et al. 1974; Buras et al. 1978) the weak neutral current mixing parameter $\theta_W$, which is related to the $g_2$ and $g_1$ couplings by

$$\sin^2\theta_W = \frac{\frac{3}{5}g_1^2}{g_2^2 + \frac{3}{5}g_1^2}, \tag{12}$$

and takes the value of $\frac{3}{8}$ in the symmetry limit where $g_2 = g_1$. The quantity (12) is renormalized at present energies because of the energy dependences $g_2(Q)$, $g_1(Q)$ of the coupling strengths mentioned earlier. The effective value of $\sin^2 \theta_W$ measured in present-day experiments should be

$$\sin^2 \theta_W^{\text{eff}} = 0.215 \pm 0.002 \tag{13}$$

for the preferred range of values of the QCD scale parameter $\Lambda$. For comparison, the present experimental average is

$$\sin^2 \theta_W^{\text{expt}} = 0.215 \pm 0.012, \tag{14}$$

when radiative corrections (Marciano & Sirlin 1981; Llewellyn Smith & Wheater 1981) are included. There is also a GUT prediction and an experimental value for the overall neutral current strength parameter $\rho(1)$, which are shown together with $\sin^2 \theta_W$ in figure 4. Clearly
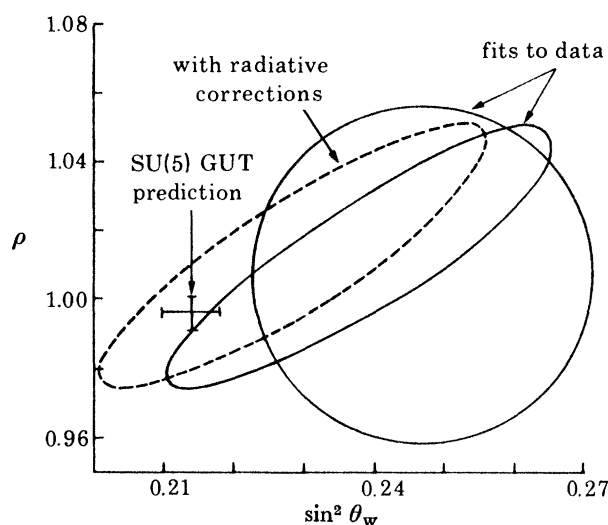


FIGURE 4. A comparison between theory and experiment for the neutral current parameters $\sin^2 \theta_W$ and $\rho$. The ellipses correspond to different ways of analysing the experimental data, while the dashed curve shows the effect of including radiative corrections in the experimental analysis.

theory and experiment are highly consistent. Other predictions of GUTs concern the ratios of quark and lepton masses (Chanowitz *et al.* 1977; Buras *et al.* 1978). These are often simple Clebsch–Gordan coefficients in the symmetry limit, and for example one expects $m_b = m_\tau$ when SU(5) is symmetric. However, the ratios are renormalized when one computes the physical masses observable in present experiments, and if one takes $m_\tau = 1.78$ GeV from experiment one deduces

$$m_b \approx 5 \text{ GeV}, \tag{15}$$

in accord with experiment. This prediction (15) is in fact valid only if there are at most six flavours of light quarks (Nanopoulos & Ross 1979) corresponding to three generations and hence to three neutrino types, in accord with the cosmological constraints (Schramm, this symposium). However, it should be emphasized that the GUT predictions analogous to (15) for the d quark mass is certainly wrong, while that for the s quark is controversial. My personal point of view is not to take these problems too seriously, though they may betoken a need to complicate our GUT.

Let us take the successes (8), (13) and (15) as evidence in favour of GUTs, and now examine their predictions for new hyperweak interactions. As has already been mentioned, the exchanges of $X$ and $Y$ bosons in figure 5 can lead to baryon decay;

$$\text{p or n} \to (\text{e}^+ \text{ or } \bar{\nu}_e) + \text{mesons } (\pi, \rho, \omega, ...) \tag{16}$$

are expected to be the dominant decays in minimal SU(5) and GUTs akin to it, with $\mu^+ + K$ suppressed by phase space and final states with mixtures of first- and second-generation particles ($e^+ + K$, $\mu^+ + \pi$, ...) suppressed by mixing angle factors. The amplitude for the exchange of
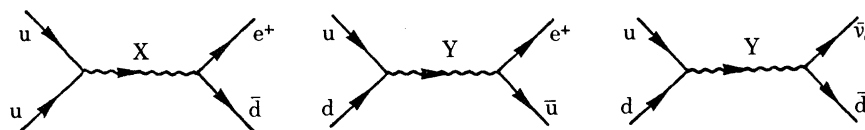


FIGURE 5. Lowest-order X and Y exchanges give an interaction that can lead to baryon decay into an antilepton and mesons.

figure 5 is proportional to $1/m_X^2$ in the same way as conventional weak amplitudes are proportional to the Fermi constant $G_F \propto 1/m_W^2$. The baryon decay rate is therefore proportional to $1/m_X^4$ and hence the nucleon lifetime

$$\tau_{p,n} = (m_X^4/m_N^5) \times O(1) ?, \tag{17}$$

where I have put in the nucleon mass factors to take care of dimensional analysis and the coefficient of $O(1)$ must be computed within some model of the baryons. The extreme sensitivity of the lifetime (17) to the superheavy boson masses should be noted: by best estimates in minimal GUTs (Ellis et al. 1980),

$$m_X = (1 \text{ to } 2) \times 10^{15} \times \Lambda, \tag{18}$$

whereas lattice QCD calculations (Hamber & Parisi 1981; Hasenfratz 1981) and recent experiments find

$$\Lambda = (0.1 \text{ to } 0.2) \text{ GeV}. \tag{19}$$

Combining the ranges (18) and (19), we find

$$m_X = (1 \text{ to } 4) \times 10^{14} \text{ GeV}, \tag{20}$$

and, incorporating reasonable calculations of the $O(1)$ coefficient in (17), we finally estimate

$$\tau_{p,n} = O(10^{27} \text{ to } 10^{31}) \text{ years.} \tag{21}$$

By now there are several experiments (Learned et al. 1979; Cherry et al. 1981) that quote lower limits on the baryon lifetime of order 1 or $2 \times 10^{30}$ years. More positively, there is an Indo-Japanese experiment (Krishnaswamy et al. 1981) with four candidate events that would correspond to a lifetime $O(10^{31})$ years if confirmed. There are at least seven rival experiments that are either running now or expecting to run soon, so we may soon know whether the Indo-Japanese experiment has been lucky, and whether the prediction (21) is correct.

Another cosmologically interesting possibility suggested by GUTs is that neutrinos may have masses. These are to be expected if lepton number is violated, for example because a Majorana mass term of the type

$$m_\nu (\Delta L = 2) \nu_L \nu_L \tag{22}$$

could no longer be forbidden. We believe that lepton number $L$ is not an exact gauge symmetry because there is no massless gauge boson analogous to the photon or gluon that couples to lepton number. Moreover, we have the 'Dogma of the Gauge Age' that the only exact symmetries are exact gauge symmetries: every global symmetry is expected to be violated at some level. Therefore, we expect in particular that lepton number will be violated, for example in transitions involving black holes,

$$\nu_{\mathrm{L}} + (\mathrm{B.H.}) \to (\mathrm{B.H.})' \to (\mathrm{B.H.}) + \bar{\nu}_{\mathrm{L}}, \tag{23}$$

on the grounds of the no-hair theorem, since lepton number is unprotected by a long-range gauge boson. In fact one does not need to go to such exotic lengths to obtain neutrino masses in non-minimal GUTs. For example, in SO(10) has one both $\nu_{\mathrm{L}} \to \nu_{\mathrm{L}}$ and $\nu_{\mathrm{L}} \to \bar{\nu}_{\mathrm{L}}$ transitions, which when diagonalized (Slansky 1979; Yanagida 1979; Barbieri *et al.* 1980*b*) provide massive neutrino eigenstates with

$$m_\nu \approx (m_1 \text{ or } m_q) \times (m_{\mathrm{W}}/m_{\mathrm{X}}) \times O(10^{0 \pm 4}), \tag{24}$$

which are much smaller than the conventional fermion masses $m_1$, $m_q$. An expected (?) range of neutrino masses may be

$$m_\nu \lesssim O(10^{-5}) \text{ eV}$$

(in minimal SU(5) supplemented by virtual black hole transitions (Barbieri *et al.* 1980*a*)) to

$$m_\nu = O(10^{+2}) \text{ eV} \tag{25}$$

(in minimal SO(10) (Witten 1980)). In general one finds that the mass of a neutrino is correlated with that of its partner charged lepton or quarks:

$$m_\nu \propto (m_1 \text{ or } m_q)^{1 \text{ or } 2}? \tag{26}$$

and the top end of the range (25) is expected to apply to the tau neutrino rather than to the electron and muon neutrinos.

Experiments on neutrino masses fall into two classes: direct measurements and indirect inferences drawn from rare decays or oscillation experiments. As far as direct measurements are concerned, most readers will be familiar with the ITEP experiment (Lyubimov *et al.* 1980*a, b*) on the end-point in tritium $\beta$ decay which suggests that

$$14 \text{ eV} < m_{\bar{\nu}_e} < 46 \text{ eV}, \tag{27}$$

a result not yet confirmed. The next best upper limit on a neutrino mass comes from internal bremsstrahlung electron capture (De Rújula 1981) on a heavy nucleus:

$$\mathrm{e}^- + (Z, A) \to (Z-1, A) + \gamma + \nu_e. \tag{28}$$

A recent experiment (Andersen *et al.* 1982) with the use of $^{163}\mathrm{Ho}$ for this process reports that

$$m_{\nu_e} < 1.3 \text{ ke V}. \tag{29}$$

Turning to indirect information, there are some discrepancies between the expected and deduced rates for double $\beta$ decay that have been interpreted as evidence for the $(\beta\beta)_{0\nu}$ process of figure 6*a* in addition to the expected $(\beta\beta)_{2\nu}$ process of figure 6*b*. The $(\beta\beta)_{0\nu}$ reaction can arise if there is a Majorana mass of the type (22) to eat up the two neutrinos of figure 6*b*. It has been suggested (Doi *et al.* 1981*a, b, c*) on the basis of the apparent discrepancies that

$$\langle m_{\nu_e} \rangle = O(30) \text{ eV}? \tag{30}$$

where the averaging in (30) refers to all the neutrino mass eigenstates weighted by their couplings to the electron. One does not expect the neutrino mass eigenstates to correspond exactly to the flavour eigenstate neutrinos, which are directly coupled to the l, μ and τ leptons. In general there will be non-trivial mixing angles that will cause a neutrino beam of a definite initial flavour to oscillate partly into other flavours as it propagates (Pontecorvo 1957, 1958; Maki *et al.* 1962; Pontecorvo 1967). Many experiments have been carried out to search for this phenomenon, but so far there is no generally accepted positive evidence. Any individual
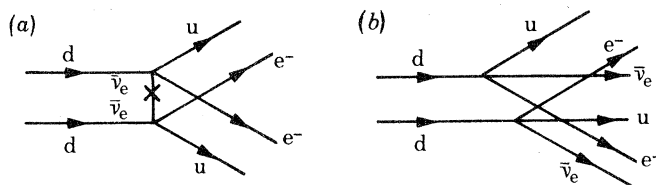


FIGURE 6. Mechanisms for (*a*) $(\beta\beta)_{0\nu}$ decay via a Majorana neutrino mass and (*b*) conventional $(\beta\beta)_{2\nu}$ decay.
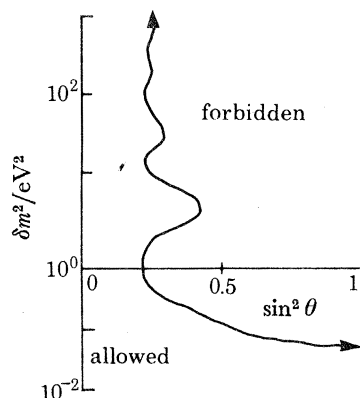


FIGURE 7. Sketch of the domains of $\delta m^2$ and $\sin^2 2\theta$ allowed and forbidden by a typical neutrino oscillation experiment.

experiment usually excludes some domain in a plane of mixing angle $\theta$ and difference in mass squared $\delta m^2$, as illustrated in figure 7. A compilation of indirect limits from experiments looking for oscillations between different neutrino flavours is shown in table 1. The quoted results correspond to limits on $\delta m^2$ if the mixing $\sin^2 2\theta$ is maximal, and on $\sin^2 2\theta$ if $\delta m^2$ is very large so that the experiment averages over many oscillations. Note that most of the limits on $\delta m^2$ are $O(1)$ eV², much less than the Russian mass (27) squared. Also the limits on mixing angles $\theta$ are often considerably smaller than the known quark charged weak mixing angle $\theta_c$: $\sin^2 \theta_c \approx 0.05$. If the Russian experiment (27) is correct, *either* the neutrino mass eigenstates are extremely degenerate in conflict with the conjectured hierarchy (26), *or* the neutrino mixing angles must be rather small.

This concludes our brief review of the structure and phenomenology of GUTs. Before going on to discuss their cosmological implications, however, it should be emphasized that, in contrast to the 'standard model' generally accepted by all particle physicists as at least approximately true, there is no such general consensus on the validity of the GUT philosophy, which should be regarded as still highly speculative.

TABLE 1. LIMITS ON NEUTRINO OSCILLATION PARAMETERS

| type of oscillation | $\delta m^2/\mathrm{eV}^2$ | $\sin^2 2\theta$ |
|---|---|---|
| $\nu_\mu \to \nu_e$ | $< 0.7$ | $< 0.002$ |
| $\nu_\mu \to \nu_\tau$ | $< 2.8$ | $< 0.027$ |
| $\nu_e \to \nu_\tau$ | $< 8$ | $< 0.6$ |
| $\bar{\nu}_\mu \to \bar{\nu}_e$ | $< 0.9$ | $< 0.2$ |
| $\bar{\nu}_\mu \to \bar{\nu}_\tau$ | $< 6.3$ | $< 0.9$ |
| $\nu_e \to$ all | $< 2.5$ | $< 0.07$ |
| $\bar{\nu}_e \to$ all | $< 0.15$ | $< 0.32$ |
| $\nu_\mu \to$ all | $< 25$ | $< 0.1$ |

## 3. BARYOSYNTHESIS

We now start our overview of the connections between GUTs and cosmology by examining the problem of generating the apparent asymmetry between matter and antimatter. Before discussing the solution proposed by GUTs (Yoshimura 1978), let us first pose the problem (Steigman 1976). The Universe contains no known concentrations of antimatter. There can be no substantial amounts of antimatter in our cluster of galaxies, or else we would have seen energetic annihilation products ($\gamma$, charged particles, etc.) coming from the interface with our own matter-dominated region. Antiprotons have been detected (Golden *et al.* 1979; Buffington *et al.* 1981) in the cosmic rays at a rate of about $10^{-4}$ of the proton flux, which is roughly consistent with their being secondary products of collisions of primary matter cosmic rays. If they had their origin in an antimatter region, one would also expect to have seen anti-$^4$He in similar proportion to the cosmic $^4$He flux, and this appears not to be so. Since no one has proposed a generally accepted mechanism for separating domains of antimatter, if they exist, beyond our local cluster, which does not in fact push them beyond the horizon, we extrapolate the inferred absence of antimatter to the whole visible Universe.

If it contains no antimatter, how much matter does the Universe contain? From one point of view, it contains very little. We recall that the microwave background radiation populates the Universe with a few hundred photons per cubic centimetre, whereas the matter observable in stars and other luminous objects amounts to about one baryon in every $10 \text{ m}^3$, and there is an upper bound on the baryon density about two orders of magnitude higher, coming from the experimental upper limit on the deceleration parameter. These direct considerations imply that

$$O(10^{-10}) \lesssim n_\mathrm{B}/n_\gamma \lesssim O(10^{-8}), \tag{31}$$

while indirect arguments based on the abundances of light elements produced during cosmological nucleosynthesis impose somewhat tighter constraints (Schramm, this symposium):

$$n_\mathrm{B}/n_\gamma = (1.5 \text{ to } 6) \times 10^{-10}, \tag{32}$$

which does not sound very much. But from another point of view the Universe contains a very large density of matter. If it had been matter–antimatter symmetric at the epoch of hadron formation and annihilation, the only relic nucleon density left after N–$\overline{\text{N}}$ annihilation would have been a small statistical fluctuation: $n_\mathrm{B}/n_\gamma \approx O(10^{-20})$. It therefore seems that the Universe must have had a small matter–antimatter asymmetry at this epoch when $T \approx 100 \text{ MeV} \approx 10^{12} \text{ K}$:

$$\frac{n_\mathrm{B} - n_{\overline{\mathrm{B}}}}{n_\mathrm{B} + n_{\overline{\mathrm{B}}}} = O(10^{-9} \text{ to } 10^{-10}). \tag{33}$$

We particle physicists would translate this baryon–antibaryon asymmetry into a quark–antiquark asymmetry when $T > 1 \text{ GeV} \approx 10^{13} \text{ K}$:

$$\frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} = O(10^{-9} \text{ to } 10^{-10}), \tag{34}$$

and our problem is to understand why and how this asymmetry arose.

The general criteria that should be met by a model of baryosynthesis were set out in a prophetic paper by Sakharov (1967). He pointed out the following.

1. One must have interactions that violate baryon number conservation. Such interactions are present in GUTs, giving rise to the anticipated baryon decays: $p, n \to (e^+, \bar{\nu}_e, \ldots) + \text{mesons}$.

2. The baryon-number violating interactions must distinguish between matter and antimatter. This means in particular that they must violate the discrete symmetry of charge conjugation C, which changes particles into antiparticles and vice versa: $C(q) = \bar{q}$, for example. If C were a good symmetry in the early Universe, then it would contain equal numbers of quarks and antiquarks. There is another discrete symmetry, parity P, which does not change the total numbers of quarks and antiquarks, but only reverses the directions of their motions and their helicities. The combined operation of CP, if it were exact, would therefore also guarantee equal densities for quarks and antiquarks. We have known for many years that the conventional weak interactions violate both C and P, as was described in §2 and figure 2. We have also known for a long time that the combination CP is also violated (Christensen *et al.* 1964). Since GUTs incorporate the weak interactions, we expect them also to violate C and CP.

3. The B, C and CP violating interactions must drop out of thermal equilibrium. This requirement is slightly more subtle. One way of understanding its necessity is to recall that the combination of discrete symmetry transformations CPT (where T stands here for time reversal) is a sacred principle of quantum field theory. In a state of thermal equilibrium, sense of the arrow of time is lost, and CPT and T symmetry in turn guarantee CP invariance and hence by the previous argument equal densities of quarks and antiquarks. In our case a breakdown of thermal equilibrium is provided by the T-violating expansion of the Universe.

To see how this occurs, we must consider the rates of GUT reactions (Dimopoulos & Susskind 1978; Ellis *et al.* 1979; Toussaint *et al.* 1979; Weinberg 1979) when $T = O(10^{15})$ GeV $= O(10^{28})$ K in the early Universe. One expects scattering $(2 \leftrightarrow 2)$ interactions mediated by vector or scalar particles to have cross sections

$$\sigma \sim \frac{\alpha^2}{T^2} \quad (T \gg m_X) \quad \to \frac{\alpha^2 T^2}{m_X^4} \quad (T \lesssim m_X) \tag{35}$$

at temperatures $T$. The corresponding interaction rates are

$$\Gamma = n(T)\sigma, \tag{36}$$

where $n(T) \sim T^3$ is the number density of essentially massless interacting particles. Comparing the interaction rates (35) and (36) with the expansion rate of the Universe, $(\dot{R}/R) \propto T^2$, we see that

$$E \equiv \left(\frac{\Gamma R}{\dot{R}}\right) \sim \frac{\alpha^2}{T} \quad (T \gg m_X) \quad \to \frac{\alpha^2 T^3}{m_X^4} \quad (T \leqslant m_X) \tag{37}$$

if the Planck scale of $10^{19}$ GeV is taken as the unit of mass. If $E$ is greater than unity we can expect to be in thermal equilibrium as far as that interaction is concerned. Curves of $E$ for

different GUT reactions (37) are plotted in figure 8, together with the rates for different decay and inverse decay $1 \leftrightarrow 2$) interactions involving heavy particles that have rates

$$\Gamma \sim \frac{\alpha m_X^2}{\sqrt{(T^2 + m_X^2)}} \frac{n_X(T)}{n_0(T)}, \tag{38}$$

where the number density $n_X(T)$ may be suppressed by a Boltzmann factor $e^{-m_X/T}$ if the species is in thermal equilibrium, or a simple factor $e^{-t/\tau}$ if it is decaying freely out of equilibrium with a lifetime $\tau$. We see from figure 8 that one expects all quantum gravitational interactions
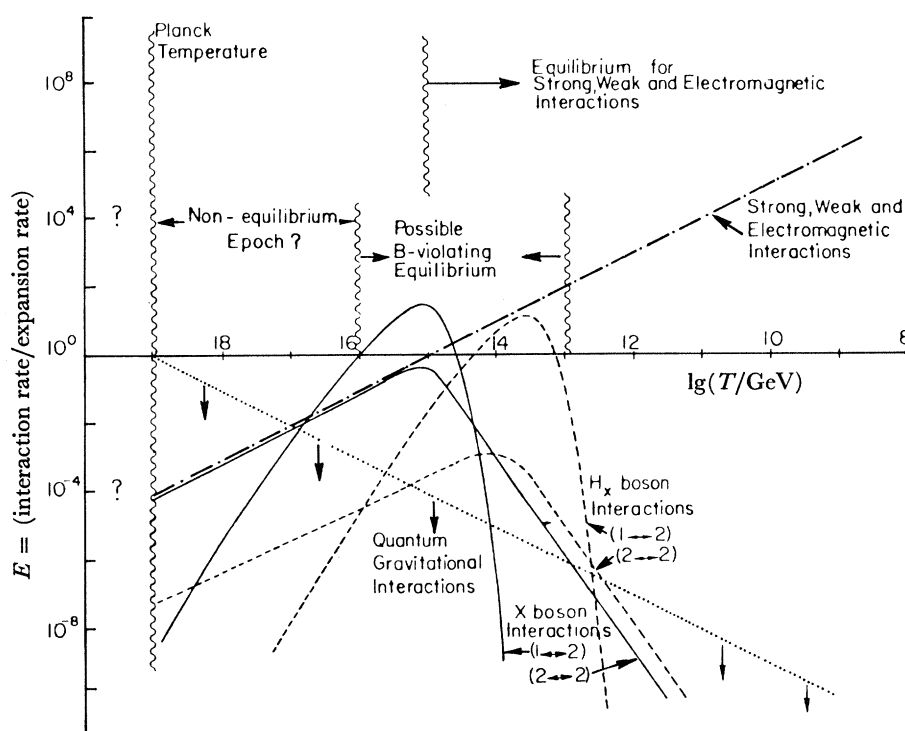


FIGURE 8. The interaction rates for the typical GUT reactions discussed in the text, compared with the expansion rate of the Universe.

to be insignificant during the epoch of interest. It is certainly possible that all grand unified particle interactions were out of equilibrium at $T > O(10^{15})$ GeV (Ellis *et al.* 1979), but likely that there would have been an equilibrium period when $T \approx O(10^{15})$ GeV. After this period, the conventional strong, weak and electromagnetic interactions would have remained in equilibrium until much later epochs, while B-violating hyperweak interactions involving heavy vector and Higgs bosons would have dropped out of equilibrium at $T \lesssim O(10^{14})$ GeV. One word of caution: while the masses and coupling strengths of vector bosons are relatively well determined, the masses, couplings and even the existence of superheavy Higgs bosons are all very uncertain. Hence the Higgs curves in figure 8 should perhaps be shifted horizontally or vertically, or even removed altogether.

 Baryosynthesis could have occurred during the period of grand unified non-equilibrium at $T \lesssim O(10^{14})$ GeV. A specific mechanism that is often favoured is the out-of-equilibrium decay of some species of superheavy particle. Superheavy gauge X and Higgs bosons $H_X$

both have competing decay qq and $\bar{q}\bar{l}$ modes. Invariance under CPT guarantees that X particles and $\overline{X}$ antiparticles (or $H_X$ and $\overline{H}_X$) have the same lifetimes:

$$\Gamma(X \to qq) + \Gamma(X \to \bar{q}\bar{l}) = \Gamma(\overline{X} \to \bar{q}\bar{q}) + \Gamma(\overline{X} \to ql). \tag{39}$$

However, their partial decay rates may differ if C and CP are violated:

$$B \equiv \frac{\Gamma(X \to qq)}{\Gamma(X \to all)} \neq \overline{B} \equiv \frac{\Gamma(\overline{X} \to \bar{q}\bar{q})}{\Gamma(\overline{X} \to all)}. \tag{40}$$

Hence, if we start off with equal densities of X and $\overline{X}$ particles, which would be guaranteed if there was indeed a period of equilibrium at $T = O(10^{15})$ GeV as suggested in figure 8, their decays will generate a net quark asymmetry:

$$\frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \approx \left(\frac{n_{X,H_X}}{n_{total}}\right) (B - \overline{B})_{X,H_X} \approx \frac{n_B}{n_\gamma}, \tag{41}$$

where ($n_X$ or $n_{H_X}/n_{total}$) is a dilution factor taking account of the possibly small number density of interesting particles X, $H_X$ relative to the total particle density $n_{total}$. In realistic GUTs one finds

$$\frac{n_B}{n_\gamma} \approx O(10^{-1} \text{ to } 10^{-2}) \, \epsilon_{X,H_X} : \epsilon_{X,H_X} \equiv (B - \overline{B})_{X,H_X}. \tag{42}$$
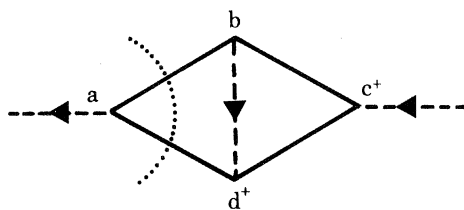


FIGURE 9. A sample lowest-order diagram that may contribute to the C- and CP-violating asymmetry in the decay of heavy Higgs bosons. The solid lines represent fermions, while the dashed lines represent Higgs particles. The dotted line picks out the physical states in the decays.

A sample lowest-order diagram which violates C and CP and can contribute to $\epsilon$ is shown in figure 9. It yields (Nanopoulos & Weinberg 1979):

$$\epsilon_{H_X} \approx \frac{\text{Im tr } (abc^\dagger d^\dagger)}{\text{tr } (aa^\dagger)}, \tag{43}$$

where $a$, $b$, $c$ and $d$ are matrices of Higgs–fermion–fermion couplings. In many models the largest CP-violating decay asymmetry comes from Higgses, with

$$\epsilon_X \approx O(\alpha/\pi) \epsilon_{H_X}. \tag{44}$$

Figure 10 shows how the q–$\bar{q}$ asymmetry can be built up in this way, starting from equal densities of X and $\overline{X}$ particles and assuming a suitable value of $\epsilon_X$ (Kolb & Wolfram 1980$a$, $b$). Given this general framework, one can now try to compute the baryon number generated in a specific GUT.

One encounters various problems when one tries to make quantitative calculations. One is that of choosing a big enough GUT. As mentioned earlier, Higgs interactions and decays often exhibit the most CP violation. Unfortunately, the lowest-order diagram of figure 9 does not contribute in the minimal SU(5) model described in §2, and one must appeal (Ellis *et al.*

1979) to the eighth-order diagram of figure 11 that gives $\epsilon_{\mathrm{H_X}} \lesssim (10^{-15})$, which is unacceptably small (cf. equations (34) and (42)). Therefore one needs a non-minimal GUT, based perhaps on another group such as SO(10). This introduces many calculational uncertainties due to unknown couplings, parameters and masses that cannot be removed until one has reliable criteria for choosing which non-minimal GUT to use.
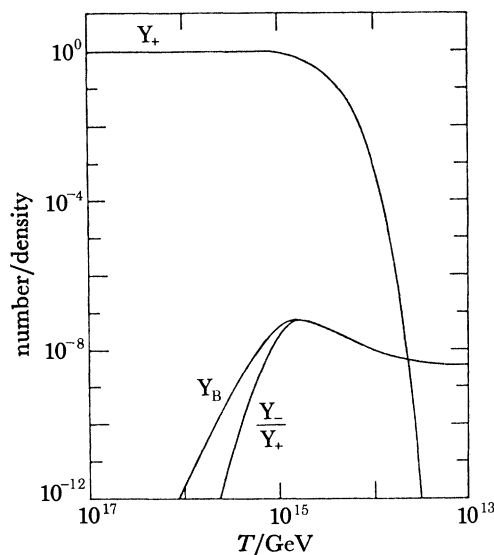


FIGURE 10. The development of particle number densities in the early Universe. $Y_+$ and $Y_-$ denote the sum and the difference of X and $\overline{X}$ particle densities relative to photons. $Y_B$ denotes the relative density of baryon number given by the difference of quark and antiquark densities. (Taken from Kolb & Wolfram (1980a, b).)
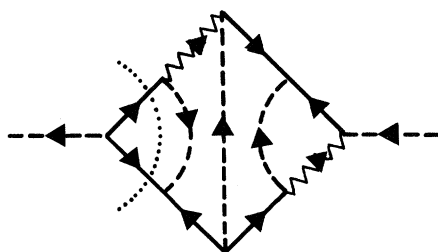


FIGURE 11. The lowest-order diagram contributing to the C- and CP-violating decays of heavy Higgs particles in the minimal SU(5) GUT. Notation as in figure 9 except that the solid lines are **10** fermions while the zigzag lines are **5** fermions.

Secondly, one should include $2 \leftrightarrow 2$ scattering interactions as well as decays. These tend (Kolb & Wolfram 1980a, b; Fry et al. 1980a, b) to wash out the generated quark excess unless $m_X$ is large enough, or the coupling $\alpha$ small enough, for the rates (37) not to overwhelm the decays. Typical calculations of this wash-out effect are shown in figure 12: they suggest that one needs

$$m_X \gtrsim O(10^{14}) \text{ GeV}; \quad \alpha \lesssim O(10^{-1}) \tag{45}$$

if one is to avoid wash-out by a factor of more than 1000. Conventional GUTs are just consistent with the constraints (45), since they have $m_X \approx (1 \text{ to } 4) \times 10^{14}$ GeV and unknown Higgs masses, while the gauge coupling strength $\alpha$ is seen from figure 12 to be about $\frac{1}{42}$, and the

Higgs couplings may be as low as $O(10^{-3})$. In view of this possibility, figure 12 gives us another reason why the baryon asymmetry may be mainly due to superheavy Higgs decays.

Thirdly, there are possible complications in the evolutionary history of the Universe. For example, perhaps thermal equilibrium was not completely established when $T \sim 10^{15}$ GeV (Ellis & Steigman 1979), so that the number densities of X and $\overline{\text{X}}$ particles could be different. Alternatively, the phase transitions expected in gauge theories may have been accompanied by supercooling and subsequent entropy generation as the Universe reheated. One such (GUT) transition is expected around the epoch of baryosynthesis (Guth, this symposium), while another should have occurred at $T \sim 10^2$ GeV when the weak interaction symmetry was broken. This latter transition is a particularly dangerous potential source of entropy (Witten 1981; Ellis et al. 1981 a, b).
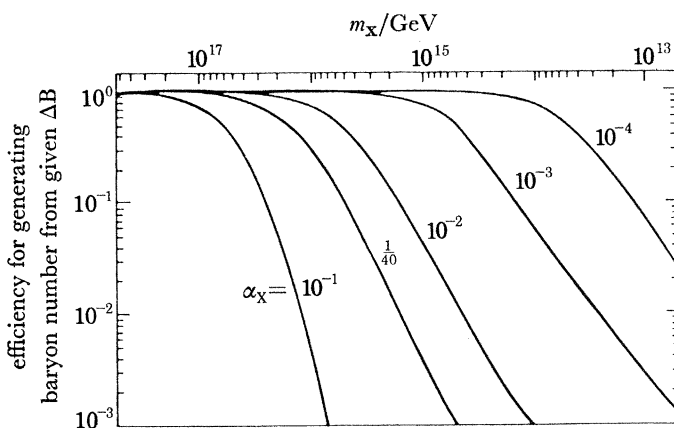


FIGURE 12. Illustration of the possible wash-out of a decay asymmetry by 2 ↔ 2 scattering interactions, illustrating the dependence on the heavy particle mass $m_X$ and coupling strength $\alpha_X$. (Taken from Kolb & Wolfram (1980 a, b).)

For these and other reasons, it seems fair to say that, while GUTs provide a natural qualitative mechanism for baryon generation, we are not yet in a position to make a reliable quantitative calculation. It is however possible in at least a class of GUTs that a particle physics observable, the neutron electric dipole moment $d_n$, may give us some semiquantitative information about baryosynthesis in the early Universe. There are contributions to $d_n$ from conventional weak interaction perturbation theory that yield (Ellis & Gaillard 1979; Gavela et al. 1982)

$$d_n = O(10^{-30\pm1}) \text{ e cm} \tag{46}$$

and from non-perturbative CP violation in QCD (Baluni 1979; Crewther et al. 1979)

$$d_n \approx 3 \times 10^{-16}\theta \text{ e cm,} \tag{47}$$

where $\theta$ is an unknown parameter characterizing the vacuum of QCD. Generally in GUTs there are contributions to $\theta$ from diagrams analogous to those responsible for baryon generation. For example, by cutting a fermion line in figure 9 and connecting up the external Higgs lines one gets figure 13, which contributes to $\theta$ and hence to $d_n$. One therefore infers a qualitative lower bound on $d_n$ in terms of $(n_B/n_\gamma)$ (Ellis et al. 1981 a):

$$\theta \gtrsim 6 \times 10^{-3} n_B/n_\gamma \Rightarrow d_n \gtrsim 2 \times 10^{-18}(n_B/n_\gamma) \text{ e cm.} \tag{48}$$

If one takes from cosmological nucleosynthesis a lower bound $(n_B/n_\gamma) \geqslant 1.5 \times 10^{-10}$ (32) (Schramm, this symposium), one deduces from (48) a lower bound

$$d_n \gtrsim 3 \times 10^{-28} \text{ e cm.} \tag{49}$$

This result is very interesting because it is much larger than the conventional weak interaction calculation (46), and not too far from the present experimental limit (Altarev *et al.* 1981):

$$d_n \lesssim 6 \times 10^{-25} \text{ e cm.} \tag{50}$$



FIGURE 13. A possible lowest-order diagram contributing to $\theta$ renormalization in a non-minimal GUT. Notation as in figure 9.

Two experimental groups are now working actively to improve this bound (50) (Ramsey 1982). If they were to find a neutron electric dipole moment in the near future, it would signal the existence of a new source of CP violation not present in the standard weak interaction model. Perhaps it would be the same source of CP violation as that responsible for our existence.

## 4. Neutrinos and cosmology

The cosmological constraints on the numbers and masses of neutrinos are of interest to grand unified theorists, and conversely the suggestions, (24) and (25), from GUTs, that neutrinos may have masses, are of interest to cosmologists and astrophysicists. Calculations of cosmological nucleosynthesis and the upper limit on the present abundance of $^4$He of 25% by mass impose a severe constraint on the number of light neutrino types (Schramm, this symposium):

$$n_\nu \lesssim 4; \quad 3 \text{ preferred.} \tag{51}$$

For comparison, the best particle physics limit on the number of neutrino types is $O(10^5)$ (Ellis 1981*b*), while we saw in §2 that the GUT calculation of the bottom quark mass only wants (Nanopoulos & Ross 1979) three light generations of light fermions corresponding to three types of light neutrino. Conventional cosmology also constrains the masses of neutrinos more severely than do particle physics experiments. One expects a large number of relic neutrinos in the present Universe:

$$\sum_\nu n_\nu \approx n_\gamma \gtrsim O(10^8) \, n_B. \tag{52}$$

If we demand that the total mass density of these ubiquitous neutrinos be less than ten times the nucleon density, we deduce from (52) that

$$\sum_\nu n_\nu \lesssim O(10^{-7}) \, m_B \approx O(100) \text{ eV.} \tag{53}$$

More sophisticated calculations do not change this limit by more than a factor of 2. For comparison, the best particle physics limits are (Ellis 1981*b*)

$$m_{\nu_e} < 50 \text{ eV}, \quad m_{\nu_\mu} < 500 \text{ keV}, \quad m_{\nu_\tau} < 200 \text{ MeV}, \tag{54}$$

with a possible indication (27) of a non-zero mass for the electron neutrino (Lyubimov *et al.* 1980*a*, *b*).

We particle physicists thank the cosmologists for their help ((51), (53)): perhaps we can offer them something in return? We saw in §2 that one generally expects (equation (25)) neutrinos in GUTs to have masses that may be as large as $O(100)$ eV. If they are heavier than 1 to 10 eV they will have dominated the formation and dynamics of galaxies and particularly galactic clusters (Cowsik & McLelland 1972; Szalay & Marx 1976; Bond *et al.* 1980). They may be able to save (Bond & Szalay 1981) the adiabatic fluctuations expected (Turner & Schramm 1979) in GUTs from conflict with experimental constraints on the isotropy of the 3 K microwave background radiation. They may seed the clustering of galaxies and provide the 'missing mass' believed to be present at various different scales in the Universe. Even oscillations between neutrinos with masses as low as $10^{-5}$ eV (equation (25)) would be sufficient to solve the solar neutrino 'problem', if indeed future experiments sensitive to lower-energy neutrinos confirm the claim (Bahcall 1978; Bahcall & Davis 1980) that fewer electron neutrinos reach the Earth than should have been produced in the Sun.

A final comment concerns the existence in many modern supersymmetric (Fayet & Ferrara 1977) versions of GUTs of 'nuinos', neutral fermionic partners of known bosons, which may be rather light. For example, the photon and the graviton should be accompanied by a photino and a gravitino. These probably decoupled from the rest of matter earlier in the Big Bang than did the neutrinos. One would therefore expect them to be less numerous in the present Universe, and they might therefore be heavier than neutrinos (equation (53)), perhaps as heavy as $O(1)$ keV. Such heavy 'nuinos' would cluster on a different, smaller, scale than conventional neutrinos and could have seeded galaxy formation (Bond *et al.* 1982; Blumenthal *et al.* 1982). It would be striking indeed if all the large-scale structures observed in the Universe were due to different light elementary particles.

## 5. THE FUTURE

We have seen how GUT interactions may have been responsible for the generation of the matter–antimatter asymmetry, and perhaps for the masses of neutrinos heavy enough to dominate galaxies: what of the future? The Universe will expand forever if the present mass density is less than the critical value of about $10^{-29}$ g cm$^{-3}$. Conversely, it will fall into another singularity if the density is above the critical value, which corresponds to a neutrino mass of order 30 eV, sitting comfortably within the range claimed (equation (27)) by the Russian experiment (Lyubimov *et al.* 1980*a, b*). If the present Hubble expansion continues in the future, we can expect protons to decay in $10^{30+n}$ years' time. The ensuing Universe will be rather drab, enlivened by the occasional black hole formation and subsequent explosion. On the other hand, perhaps the Russian experiment is correct and the neutrino mass dooms us to a closed Universe, a sort of cosmic gulag, and eventual collapse into an anti-Big Bang. In this case, perhaps we will meet again in recycled form to debate these issues in another incarnation of the Royal Society, $10^{11}$ years hence.

## REFERENCES

Altarev, I. S. *et al.* 1981 *Phys. Lett.* B **102**, 13.

Anderson, J. V. *et al.* 1982 CERN preprint no. EP/82–50.

Bahcall, J. N. 1978 *Rev. mod. Phys.* **50**, 88.

Bahcall, J. N. & Davis, R. 1980 Contribution to the *Festschrift for Willy Fowler.*

Baluni, V. 1979 *Phys. Rev.* D **19**, 2227.

Barbieri, R., Ellis, J. & Gaillard, M. K. 1980*a* *Phys. Lett.* B **90**, 249.

Barbieri, R., Nanopoulos, D. V., Morchio, G. & Strocchi, F. 1980*b* *Phys. Lett.* B **90**, 91.

Blumenthal, G. R., Pagels, H. & Primack, J. 1982 U.C. Santa Cruz preprint no. UCSC-TH/146/82.

Bond, J. R., Efstathiou, G. & Silk, J. 1980 *Phys. Rev. Lett.* **45**, 1980.

Bond, J. R. & Szalay, A. S. 1981 In *Proc. Neutrino '81 (High Energy Physics Group, Hawaii)*, p. 59.

Bond, J. R., Szalay, A. S. & Turner, M. S. 1982 *Phys. Rev. Lett.* **48**, 1636.

Buffington, A., Schindler, S. M. & Pennypacker, C. R. 1981 *Astrophys. J.* **248**, 1179.

Buras, A. J., Ellis, J., Gaillard, M. K. & Nanopoulos, D. V. 1978 *Nucl. Phys.* B **135**, 66.

Chanowitz, M. S., Ellis, J. & Gaillard, M. K. 1977 *Nucl. Phys.* B **128**, 506.

Cherry, M. L. *et al.* 1981 *Phys. Rev. Lett.* **47**, 1507.

Christensen, J. M., Cronin, J. W., Fitch, V. L. & Turlay, R. 1964 *Phys. Rev. Lett.* **13**, 138.

Cowsik, R. & McLelland, J. 1972 *Phys. Rev. Lett.* **29**, 669.

Crewther, R. J., Di Vecchia, P., Veneziano, G. & Witten, E. 1979 *Phys. Lett.* B **88**, 123.

De Rújula, A. 1981 *Nucl. Phys.* B **188**, 414.

Dimopoulos, S. & Susskind, L. 1978 *Phys. Rev.* D **18**, 4500.

Doi, M. *et al.* 1981*a* *Phys. Lett.* B **103**, 219.

Doi, M. *et al.* 1981*b* *Prog. theor. Phys.* **66**, 1739.

Doi, M. *et al.* 1981*c* *Prog. theor. Phys.* **66**, 1765.

Ellis, J. 1981*a* In *Gauge theories and experiments at high energies* (ed. K. C. Bowler & D. G. Sutherland) (Scottish Universities Summer School in Physics, Edinburgh), p. 201.

Ellis, J. 1981*b* CERN preprint no. TH-3174/LAPP-TH-48.

Ellis, J. & Gaillard, M. K. 1979 *Nucl. Phys.* B **150**, 141.

Ellis, J., Gaillard, M. K. & Nanopoulos, D. V. 1979 *Phys. Lett.* B **88**, 320.

Ellis, J., Gaillard, M. K., Nanopoulos, D. V. & Rudaz, S. 1980 *Nucl. Phys.* B **176**, 61.

Ellis, J., Gaillard, M. K., Nanopoulos, D. V. & Rudaz, S. 1981*a* *Nature, Lond.* **293**, 41.

Ellis, J., Gaillard, M. K., Nanopoulos, D. V. & Rudaz, S. 1981*b* *Phys. Lett.* B **99**, 101.

Ellis, J., Gaillard, M. K., Nanopoulos, D. V. & Rudaz, S. 1981*c* *Phys. Lett.* B **106**, 298.

Ellis, J. & Nanopoulos, D. V. 1981 *Nature, Lond.* **292**, 436.

Ellis, J. & Steigman, G. 1979 *Phys. Lett.* B **84**, 186.

Fayet, P. & Ferrara, S. 1977 *Phys. Rep.* **32**, 249.

Froggatt, C. D. & Nielsen, H. B. 1979 *Nucl. Phys.* B **164**, 114.

Fry, J. N., Olive, K. A. & Turner, M. S. 1980*a* *Phys. Rev.* D **22**, 2953.

Fry, J. N., Olive, K. A. & Turner, M. S. 1980*b* *Phys. Rev.* D **22**, 2977.

Gavela, M. B. *et al.* 1982 *Phys. Lett.* B **109**, B 215.

Gell-Mann, M. & Low, F. E. 1954 *Phys. Rev.* **95**, 1300.

Georgi, H. 1975 In *Particles and fields* – 1974 (ed. C. E. Carlson), p. 575. New York: A.I.P.

Georgi, H. & Glashow, S. L. 1974 *Phys. Rev. Lett.* **32**, 438.

Georgi, H., Quinn, H. R. & Glashow, S. L. 1974 *Phys. Rev. Lett.* **33**, 451.

Glashow, S. L. 1961 *Nucl. Phys.* **22**, 579.

Golden, R. L. *et al.* 1979 *Phys. Rev. Lett.* **43**, 1196.

Gross, D. J. & Wilczek, F. A. 1973 *Phys. Rev. Lett.* **30**, 1343.

Hamber, H. & Parisi, G. 1981 *Phys. Rev. Lett.* **47**, 1792.

Hasenfratz, P. 1981 In *Proc. 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn* (ed. W. Pfeil), p. 866. Bonn: Physikalisches Institut.

Kim, J. E., Langacker, P., Levine, M. & Williams, H. H. 1981 *Rev. mod. Phys.* **53**, 211.

Kobayashi, M. & Maskawa, T. 1973 *Prog. theor. Phys.* **49**, 652.

Kolb, E. W. & Wolfram, S. 1980*a* *Phys. Lett.* B **91**, 217.

Kolb, E. W. & Wolfram, S. 1980*b* *Nucl. Phys.* B **172**, 224.

Krishnaswamy, M. R. *et al.* 1981 *Phys. Lett.* B **106**, 339.

Langacker, P. 1981*a* *Phys. Rep.* C **72**, 185.

Langacker, P. 1981*b* In *Proc. 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn* (ed. W. Pfeil), p. 823. Bonn: Physikalisches Institut.

Learned, J., Reines, F. & Soni, A. 1979 *Phys. Rev. Lett.* **43**, 907.

Liede, I. & Roos, M. 1980 *Nucl. Phys.* B **167**, 397.

Llewellyn Smith, C. H. & Wheater, J. F. 1981 *Phys. Lett.* B **105**, 486.

Lyubimov, V. A. *et al.* 1980*a*  *Yad. Fiz.* **32**, 30.

Lyubimov, V. A. *et al.* 1980*b*  *Phys. Lett.* B **94**, 266.

Maki, Z., Nakagawa, M. & Sakata, S. 1962  *Prog. theor. Phys.* **28**, 870.

Marciano, W. J. & Pagels, H. 1978  *Phys. Rep.* **36**, 137.

Marciano, W. J. & Sirlin, A. 1981  *Nucl. Phys.* B **189**, 442.

Nanopoulos, D. V. 1980  In *Proc. École d'été de physique des particules, Gif-sur-Yvette* (IN2P3, Paris), p. 1.

Nanopoulos, D. V. & Ross, D. A. 1979  *Phys. Rev.* D **20**, 2484.

Politzer, H. D. 1973  *Phys. Rev. Lett.* **30**, 1346.

Pontecorvo, B. 1957  *Zh. éksp. teor. Fiz.* **33**, 549.

Pontecorvo, B. 1958  *Zh. éksp. teor. Fiz.* **34**, 247.

Pontecorvo, B. 1967  *Zh. éksp. teor. Fiz.* **53**, 1717.

Ramsey, N. F. 1982  *Comments nucl. Particle Phys.* **10**, 227.

Sakharov, A. D. 1967  *Pis'ma Zh. éksp. teor. Fiz.* **5**, 32.

Salam, A. 1968  In *Proc. 8th Nobel Symposium, Stockholm* (ed. N. Svartholm), p. 367. Stockholm: Almqvist & Wiksells.

Slansky, R. 1979  Talk at Sanibel Symposium, Caltech preprint no. CALT-68-709.

Steigman, G. 1976  *A. Rev. Astr. Astrophys.* **14**, 339.

Stueckelberg, E. C. G. & Peterman, A. 1953  *Helv. phys. Acta* **26**, 499.

Szalay, A. S. & Marx, G. 1976  *Astron. Astrophys.* **49**, 437.

Toussaint, D., Treiman, S. B., Wilczek, F. A. & Zee, A. 1979  *Phys. Rev.* D **19**, 1036.

Turner, M. S. & Schramm, D. N. 1979  *Nature, Lond.* **279**, 303.

Weinberg, S. 1967  *Phys. Rev. Lett.* **19**, 1264.

Weinberg, S. 1979  *Phys. Rev. Lett.* **42**, 850.

Witten, E. 1980  *Phys. Lett.* B **91**, 81.

Witten, E. 1981  *Nucl. Phys.* B **177**, 477.

Yanagida, T. 1979  In *Proc. Workshop on the Unified Theory and the Baryon Number in the Universe* (KEK, Japan).

Yoshimura, M. 1978  *Phys. Rev. Lett.* **41**, 381.